Consistent Conjectures In A Human Migration Model

Vyacheslav V. Kalashnikov, Instituto Tecnológico y de Estudios México
Nataliya I. Kalashnykova, Universidad Autónoma de Nuevo León México
Lillian R. Chávez Delgadillo, Instituto Tecnológico y de Estudios Superiores

Abstract

In this paper, we develop a human migration model using the concept of a conjectural variations equilibrium (CVE). In contrast to previous works we extend the model to the case where the conjectural variations coefficients may be not only constants, but also (continuously differentiable) functions of the total population at the destination and of the group's fraction in it. Moreover, we allow these functions to take distinct values at the abandoned location and at the destination. As an experimental verification of the proposed model, we develop a specific form of the model based upon relevant population data of a three-city agglomeration at the boundary of two Mexican states: Durango (Dgo.) and Coahuila (Coah.) Namely, we consider the 1980 – 2005 dynamics of population growth in the three cities: Torreón (Coah.), Gómez Palacio (Dgo.) and Lerdo (Dgo.), and propose utility functions of four various kinds for each of the three cities. After having collected necessary information about the average movement and transportation costs for each pair of the cities, we apply the above-mentioned human migration model to this example. Also, we introduce three different concepts of consistent conjectures, namely: strongly-, weakly-, and mixed-consistent conjectures, and establish the existence results.

Keywords: Human migration model, variational inequality formulation, conjectural variations equilibrium, strongly, weakly, and mixed-consistent conjectures.

1. Introduction

Migration problems are vivid and important for many countries throughout the world. As a response of researchers to that problem, various migration theories have been developed since the eighteenth century. In the short historical review of these theories, we mainly rely on the brilliant survey by Akkoyunlu and Vickerman (2000), who distinguish the following trends.

The traditional model of migration is the ***neo-classical/labour-flow approach***, in which migration is viewed as a response to regional market disequilibrium. For example, a regional wage differential may induce a migration flow from low wage to high wage regions. In equilibrium, production factors receive the same real return in any region, thus producing a narrowing of real wage differentials (Smith, 1776; Ravenstein, 1889; Hicks, 1932). Other papers developing the labour-flow models consider also regional differences in skill or education (*cf*. Daly and Ghatak, 1999). Various models used *expected* rather than actual earning differentials as the determinant of migration, and introduced further factors such as the costs of moving, differences in the cost of living, occupational structure (Harris and Todaro, 1970; Aghion and Blanchard, 1994; Hatton, 1995). The main disadvantage of the labour-flow models, however, is that it is difficult for these models to explain differences in migration behaviour for what otherwise appears to be comparable individuals (*cf*. Akkoyunlu and Vickerman, 2000).

Developing a different scheme, ***human capital models*** treat migration as an investment decision in which individuals calculate their present discounted value of expected returns in every location (Becker, 1962; Schultz, 1961; Sjaastad, 1962). Empirical modelling of flows relates the expected present value of a move to individual characteristics, such as age, education, occupation, employment status, sex, race, marital status, and family size, of which the following two are particularly important. Younger persons are expected to be more mobile than older persons, because they can expect more years of enjoying the benefits received from migration. Education determines general human capital which is easily transferable to different locations and reduces the risks of migration. However, there are minor points in the human capital model. A move is normally viewed as an irrevocable lifetime decision and, in concentrating on income, the model ignores many non-pecuniary aspects of a move, differences in the consumption activities of the potential migrant and financing the move (*see* Akkoyunlu and Vickerman, 2000).

Later, in 1970s, ***household migration models*** became quite popular (*cf*. Akkoyunlu and Vickerman, 2000). For instance, Mincer (1978) showed that increased female labour force participation rates lead to increased interdependence with the partner's migration decision, which results in less migration. Those family members who do not move of their own initiative often have to expect reduced earnings and fewer employment opportunities in the labour market of the destination country. Therefore, a family will only migrate if the gains of one family member internalise the losses of other family members (Sandell, 1977). On the other hand, Stark (1991) considers migration as risk-sharing behaviour within families. The family aims at maximizing income, but also at minimizing risks to the family income. The latter is achieved by some members of the family working in labour markets with wages and employment conditions negatively (or weakly) correlated with those in the home region. This hedging strategy helps a family to secure their income in the case of an economic collapse in local labour market via remittances from family members working abroad (Poirine, 1997). Moreover, as migration of some members may improve the relative income of the household in the home region, hence not only income differentials between regions, but also the income distribution in the original location, affects the migration decision (Stark and Taylor, 1989).

Recently, a new branch of migration model, namely ***networks models***, has been studied by a series of authors (*cf*. Akkoyunlu and Vickerman, 2000). There is evidence of strong flows between certain parts of countries and not others, and that immigrants from one country often cluster in specific cities in the host country. This proves the importance of networks that link new migrants by ties of ethnicity, kinship and friendship (Bauer and Zimmermann, 1997). Migration becomes a self-sustaining process because the costs and risks of migration for subsequent migrants are lowered by social and informational networks. Rising wages in the home country and falling wages in the receiving country weakens the self-perpetuating process and lowers the possible benefits of moving (Hugo, 1981; Massey, 1990a, 1990b; Massey and Espana, 1987).

As well as network ties there may be other external influences, such as differences in the range of regional amenities (*see* Akkoyunlu and Vickerman, 2000). These include public or merit goods, such as educational opportunities, health care systems and general living conditions, or externalities in consumption via the variety and kinds of market goods available. All these items could be represented by certain functions called ***utility functions***. As a society becomes more prosperous, regional amenities replace pecuniary motives in the migration process; such as the family decentralizing moves from large cities. As consumers move to a region to purchase amenities, land rents may rise until households are in equilibrium. Hence, migration is seen as *equilibrating* both the labour market and the land market.

Because of that, human migration ***equilibrium*** models have been in the centre of research activity since the early nineties of the last century (*cf*. Nagurney 1989, 1990, Nagurney et al. 1992, Nagurney 1999, among others). In almost all relevant papers and books, a network of locations is considered, and conditions guaranteeing the existence and uniqueness of equilibrium in the proposed models are elaborated. For example, the works by the group of Anna Nagurney cited above examined various forms of the Nash equilibrium under an assumption of perfect competition, that is, each population group neglected the possible influence of the migration flows on the living standards at the destination.

In the papers by Bulavsky and Kalashnikov (1994, 1995), Isac et al. (2002), a new array of conjectural variations equilibria (CVE) was introduced and investigated, in which the influence coefficients of each agent affected the structure of the Nash equilibrium. In particular, constant conjectural influence factors were used in the human migration model examined in Isac et al. (2002). More precisely, the potential migration groups were taking into account not only the current difference between the utility function values at the destination and original locations, but also the possible variations in the utility values implied by the change of population volume due to the migration flow. In other words, we considered not a perfect competition but a generalized Cournot-type model with influence coefficients in general different from 1 (as it is in the classical Cournot model).

In this paper, we extend the latter model to the case where the conjectural variations coefficients may be not only constants, but also functions of the total population at the destination and of the group's fraction in it. Moreover, we allow these functions to take distinct values at the abandoned location and at the destination, which should elevate the model's flexibility. In contrast to the well-known existence and uniqueness results cited as Theorems 3.1 and 3.2, the existence and uniqueness Theorems 3.3 and 3.4 are original ones. As an experimental verification of the proposed model, we develop a specific form of the model based upon relevant population data of a three-city agglomeration at the boundary of two Mexican states: Durango (Dgo.) and Coahuila (Coah.) Namely, we consider the 1980-2000 dynamics of population growth in the three cities: Torreón (Coah.), Gómez Palacio (Dgo.) and Lerdo (Dgo.), and propose utility functions of three various kinds for each of the three cities. To our knowledge, utility functions of these types were not used in the previous literature dealing with the human migration model. After having collected necessary information about the average movement and transportation (*i.e.*, migration) costs for each pair of the cities, we apply the above-mentioned human migration model to this example. Numerical experiments have been conducted with interesting results concerning the probable equilibrium states revealed.

The paper is organized as follows. The next section (Section 2) describes the proposed human migration model and introduces the appropriate notation. Section 3 is dedicated to the definition of the conjectural variations equilibrium in the model. In Section 4, Theorems 4.1 and 4.2 are obtained which establish the conditions guaranteeing the existence and uniqueness of the human migration equilibrium with conjectural variations (CVE). Definition of three different kinds of the consistency concept for the influence coefficients (conjectures), together with the results of the existence of both consistent conjectures and equilibria are presented in Section 5. Conclusions (Section 6) and acknowledgements complete the paper.

2. The Model

Similar to Isac et al. (2002), consider a closed economy with:

n locations, denoted by i

K classes of population, denoted by k

$\overline{Q}_i^k$ initial fixed population of class k in location i

Q_i^k final population of class k in location i

c_{ij} cost of migration from location i to location j

s_{ij}^k migration flow of class k from origin i to destination j

Assume that the migration cost reflects not only the cost of physical movement but also the personal and psychological cost as perceived by a class when moving between locations.

Unlike the model of human migration described by Isac et al. (2002), the utility u_i^k (attractiveness of location i as perceived by class k), depends on the population at destination Q_i^k, that is, $u=u(Q)$. This assumption is quite natural: indeed, in many cases, the cities with higher population provide for much more possibilities to find a job, better medical service and household facilities, a developed infrastructure etc. On the other hand, when the infrastructure development lags behind the modern city demands, the higher population may lead to certain decrease in the living standards, and hence, of the utility values.

The conservation of flow equations, given for each class k and each location i and assuming no repeated or chain migration, are given as follows:

$$Q_i^k = \overline{Q}_i^k + \sum_{j\neq i} s_{ji}^k - \sum_{j\neq i} s_{ij}^k, \qquad i=1,\dots,n, \tag{2.1}$$

and

$$\sum_{j \neq i} s_{ij}^k \leq \overline{Q}_i^k, \quad i = 1,...,n, \tag{2.2}$$

with $s_{ij}^k \geq 0, \forall k = 1,\dots,K; j \neq i$. Denote the problem's feasible set by

$$M = \left\{ (Q,s) \middle| s \geq 0, \ (Q,s) \ \text{satisfies (2.1), (2.2)} \right\} .$$

Equation (2.1) states that the population of class *k* at location *i* is determined by the initial population of class *k* at location *i* plus the migration flow into *i* of that class minus the migration flow out of *i* for that class. Equation (2.2) affirms that the flow out of *i* by class *k* cannot exceed the initial population of class *k* at *i*, since no chain migration is allowed.

Assume that migrants are rational, and that migration continues until no individual has any incentive to move, since a unilateral decision will no longer yield a positive net gain (the gain in the expected utility minus the migration cost).

In order to extend the human migration model by Isac et al. (2002), here we introduce the following concepts.

Let $w_{ij}^{k+} \geq 0$ be an influence coefficient taken in account by an individual of class *k* moving from *i* to *j*. This coefficient is defined by her assumption that after the movement of s_{ij}^k individuals of class *k* from *i* to *j* the total population of class *k* at *j* will become equal to:

$\overline{Q}_j^k + w_{ij}^{k+} s_{ij}^k$.

On the other hand, let $w_{ij}^{k-} \geq 0$ be an influence coefficient conjectured by an individual of class *k* moving from *i* to *j*, determined by the assumption that after the movement of s_{ij}^k individuals, the total population of class *k* in *i* will remain equal to

$\overline{Q}_i^k - w_{ij}^{k-} s_{ij}^k$.

We accept the following assumptions concerning the utility functions and expected variations of the utility values:

A1. The utility $u_i^k = u_i^k(Q_i^k)$ is a monotone decreasing and continuously differentiable function.

A2. Each person of class *k*, when considering her possibility of moving from location *i* to location *j*, takes into account not only the difference in the utility values at the initial location and the destination, but also both the expected (negative) increment of the utility function value at *j*

$$s_{ij}^k w_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k}$$

and the expected (positive) utility value increment in location *i*

$$- s_{ij}^k w_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} .$$

3. Definition of Equilibrium

A multi-class population and flow pattern $(Q^*, s^*) \in M$ is an equilibrium, if for each class $k = 1,\dots,K$, and for each pair of locations $i, j = 1,\dots,n; i \neq j$, the following relationship holds:

$$u_i^k - s_{ij}^{k*} w_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k}\left(Q^*\right) + c_{ij}^k \begin{cases} = u_j^k + s_{ij}^{k*} w_{ij}^{k+} \dfrac{\partial u_j^k}{\partial Q_j^k}\left(Q^*\right) - \lambda_i^k, \text{ if } \; s_{ij}^{k*} > 0; \\ \geq u_j^k + s_{ij}^{k*} w_{ij}^{k+} \dfrac{\partial u_j^k}{\partial Q_j^k}\left(Q^*\right) - \lambda_i^k, \text{ if } \; s_{ij}^{k*} = 0; \end{cases} \tag{3.1}$$

and

$$\lambda_i^k \begin{cases} \geq 0, \text{ if } \; \sum_{l \neq i} s_{il}^{k*} = \overline{Q}_i^k; \\ = 0, \text{ if } \; \sum_{l \neq i} s_{il}^{k*} < \overline{Q}_i^k. \end{cases} \tag{3.2}$$

A3. We assume that the influence coefficients are functions depending upon the current population at the location in question and the migration flow from location *i* to location *j*, satisfying the following conditions:

$s_{ij}^k w_{ij}^{k+}(Q,s) = \alpha_{ij}^{k+} s_{ij}^k + \sigma_{ij}^{k+} Q_j^k$,

and

$s_{ij}^k w_{ij}^{k-}(Q,s) = \alpha_{ij}^{k-} s_{ij}^k - \sigma_{ij}^{k-} Q_i^k$,

where

$\alpha_{ij}^{k\pm} \geq 0, \; \sigma_{ij}^{k\pm} \geq 0, \; k = 1,\ldots,J; \;\; i \neq j.$

.

This turns (3.1) into:

$$u_i^k - s_{ij}^{k*} \alpha_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} + \sigma_{ij}^{k-} Q_i^{k*} \frac{\partial u_i^k}{\partial Q_i^k} + c_{ij}^k = $$

$$= u_j^k + s_{ij}^{k*} \alpha_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} + \sigma_{ij}^{k+} Q_j^{k*} \frac{\partial u_j^k}{\partial Q_j^k} - \lambda_i^k, \text{ if } \; s_{ij}^{k*} > 0; \tag{3.3a}$$

and

$$u_i^k - s_{ij}^{k*} \alpha_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} + \sigma_{ij}^{k-} Q_i^{k*} \frac{\partial u_i^k}{\partial Q_i^k} + c_{ij}^k \geq $$

$$\geq u_j^k + s_{ij}^{k*} \alpha_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} + \sigma_{ij}^{k+} Q_j^{k*} \frac{\partial u_j^k}{\partial Q_j^k} - \lambda_i^k, \text{ if } \; s_{ij}^{k*} = 0. \tag{3.3b}$$

Now assume that the utility function associated with a particular location and a single class can depend upon the population associated with every class and each location, that is, compose a vector-function $u = u(Q)$. Assume also that the cost associated with migration between two locations as perceived by a particular class can depend, in general, upon the flow of each class between every pair of locations, i.e. compose an aggregate vector-function $c = c(s)$. Finally, let us compose an auxiliary vector of the appropriate size as follows:

$$s_{ij}^{k}\alpha_{ij}^{k-}\frac{\partial u_i^k}{\partial Q_i^k} - \sigma_{ij}^{k-}Q_i^k\frac{\partial u_i^k}{\partial Q_i^k} + s_{ij}^{k}\alpha_{ij}^{k+}\frac{\partial u_j^k}{\partial Q_j^k} - \sigma_{ij}^{k+}Q_j^k\frac{\partial u_j^k}{\partial Q_j^k} = d_{ij}^k(Q,s);$$

$$d(Q,s) = \left(d_{ij}^k(Q,s)\right).$$

4. Existence and Uniqueness of Equilibrium

Now we are in a position to formulate the following result, established in the previous paper:

Theorem 4.1 **[*Kalashnikov and Kalashnykova, 2006*].**

A population and migration flow pattern $(Q^*, s^*) \in M$ *satisfies the equilibrium conditions* (3.1) *and* (3.2) *if, and only if it solves the variational inequality problem*

$$\langle -u(Q^*), Q - Q^* \rangle + \langle c(s^*) - d(Q^*, s^*), s - s^* \rangle \geq 0, \forall (Q,s) \in M . \tag{4.1}$$

■

The existence of at least one solution to the variational inequality (4.1) follows from the general theory of variational inequalities, under the sole assumption of continuous differentiability of the utility functions *u* and continuity of migration cost functions *c*, since the feasible convex set *M* is compact (*cf*., for example, Kinderlehrer and Stampacchia , 1980).

From now on, we omit the superscript *k* for simplicity purpose. The uniqueness of the equilibrium population and migration flow pattern (Q^*, s^*) follows under the assumption that the compound operator

$$\begin{pmatrix} -u(Q) \\ c(s) - d(Q,s) \end{pmatrix} : R^{K\times n} \times R^{K\times n\times(n-1)} \to R^{K\times n} \times R^{K\times n\times(n-1)},$$

involving the utility and migration cost functions, is strictly monotone over the feasible set *M*:

$$\left\langle \begin{pmatrix} -u(Q^1) \\ c(s^1) - d(Q^1, s^1) \end{pmatrix} - \begin{pmatrix} -u(Q^2) \\ c(s^2) - d(Q^2, s^2) \end{pmatrix}, \begin{pmatrix} Q^1 - Q^2 \\ s^1 - s^2 \end{pmatrix} \right\rangle > 0, \quad \forall \begin{pmatrix} Q^1 \\ s^1 \end{pmatrix} \neq \begin{pmatrix} Q^2 \\ s^2 \end{pmatrix},$$

that is,

$$-\left\langle u(Q^1) - u(Q^2), Q^1 - Q^2 \right\rangle + \left\langle c(s^1) - c(s^2), s^1 - s^2 \right\rangle - \left\langle d(Q^1, s^1) - d(Q^2, s^2), s^1 - s^2 \right\rangle > 0 . \tag{4.2}$$

The latter is a consequence of the classical result of the Theory of Variational Inequality Problems (*cf*., for example, Kinderlehrer and Stampacchia, 1980):

Theorem 4.2. *Consider the variational inequality: Find* $y^* \in M \subset R^n$ *such that,*

$$\langle F(y^*), y - y^* \rangle \geq 0, \quad \forall y \in M. \tag{4.3}$$

If the operator $F : R^n \to R^n$ *is strictly monotone, that is,*

$$\langle F(y^1) - F(y^2), y^1 - y^2 \rangle > 0, \qquad \forall y^1 \neq y^2,$$

then the variational inequality (3.3) *has at most one solution.* ∎

5. Consistent conjectures

The consistency of conjectures (or, the influence coefficients) arises naturally as an important issue. Indeed, the existence of at least one equilibrium for arbitrary influence coefficients obliges one to select some justified conjectures so that the above concept of the equilibrium make sense. In this section, we propose three different concepts of the consistency and formulate the existence results for each of them.

Definition 5.1. We say that the conjectures (influence coefficients) $w^{k+} = \left(w_{ij}^{k+}\right), k = 1, \ldots, K$, and the corresponding conjectural variations equilibrium (CVE) $\left(Q^*, s^*\right) \in M$, are ***strongly consistent*** if, and only if, the following equalities are valid:

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} s_{ij}^{k*} - \sum_{\substack{m=1 \\ m \neq j}}^{n} s_{jm}^{k*} = w_{1j}^{k+} s_{1j}^{k*} = \cdots = w_{j-1,j}^{k+} s_{j-1,j}^{k*} = \cdots = w_{nj}^{k+} s_{nj}^{k*},\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, K. \tag{5.1}$$

The conjectures are called ***mixed-consistent***, if for each $j = 1, 2, \ldots, n$, the following relationships hold:

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} s_{ij}^{k*} - \sum_{\substack{m=1 \\ m \neq j}}^{n} s_{jm}^{k*} = \frac{1}{\sum_{i \neq j} w_{ij}^{k+}} \sum_{i \neq j} w_{ij}^{k+} s_{ij}^{k*}, \quad k = 1, 2, \ldots, K. \tag{5.2}$$

Finally, we refer to the conjectures as to ***weakly consistent***, if for all $j = 1, 2, \ldots, n$, one has

$$\sum_{k \in K} \left(\sum_{\substack{i=1 \\ i \neq j}}^{n} s_{ij}^{k*} - \sum_{\substack{m=1 \\ m \neq j}}^{n} s_{jm}^{k*} \right) = \sum_{k \in K} \left(\frac{1}{\sum_{i \neq j} w_{ij}^{k+}} \sum_{i \neq j} w_{ij}^{k+} s_{ij}^{k*} \right). \tag{5.3}$$

Consistency of the conjectures $w^{k-} = \left(w_{ij}^{k-}\right), k = 1, \ldots, K$, is defined in an analogous manner.

Now we can state the following existence result based upon Fixed Point Theory.

Theorem 5.2. *Under assumptions* A1 – A3, *there exist sets of strongly, weakly, and mixed-consistent conjectures (influence coefficients)* $w^{k\pm} = \left(w_{ij}^{k\pm}\right), k = 1, \ldots, K$, *together with the corresponding consistent CVE* $\left(Q^*, s^*\right) \in M$. ∎

6. Conclusions

We have investigated a human migration model involving conjectures of the migration groups concerning the variations of utility function values both in the abandoned location and in the destination site. To formulate equilibrium conditions in this model, we use the concept of conjectural variation equilibrium (CVE). We establish the existence and uniqueness results for the equilibrium in question, and introduced concepts of strongly, weakly, and mixed-consistent conjectures (influence coefficients), together with the corresponding CVEs. The theorem guaranteeing the existence of each of the above types of consistent conjectures and the conjectural variations equilibrium, has been also proved.

We also note that the human migration model with conjectural variations can be further extended and examined in the case when constraint (1.2) is replaced by a weaker condition, say

$$Q_i^k \geq 0, \tag{6.1}$$

that allows us to consider the ***repeated*** or ***chain*** migration. In this case the feasible set K stops being compact (remaining convex, however), which makes insufficient the use of the general theory of variational inequality problems to demonstrate the existence of equilibrium. Then subtler results obtained by Bulavsky, Isac and Kalashnikov (1998) and further developed in Isac, Bulavsky, and Kalashnikov (2002), can be used to that effect. Indeed, the existence of equilibrium will be guaranteed for ***various classes*** of utility functions and migration costs that are free of exceptional families of elements (EFE).

Acknowledgements

The first and the third authors' research activities were financially supported by the CONACyT's program "SNI Tutorial Professional", project No. 104914. The first and the second authors were also supported by the SEP-CONACYT grant No. CB-2008-01-106664, and by the Russian Foundation for Humanity Research project RGNF 08-02-00271.

References

1. Aghion, P., and Blanchard, O.J. (1994), On the speed of transition in Central Europe, *Discussion Papers in Economics*, 87, Nuffield College, Oxford.
2. Akkoyunlu, S., and Vickerman, R. (2001), Migration and the efficiency of European labour markets, Working Paper, Department of Economics, The University of Kent at Canterbury.
3. Bauer, T., and Zimmermann, K.F. (1997), Network migration of ethnic Germans, *International Migration Review* 31: 143 – 149.
4. Becker, G.S. (1962), Investments in human capital: A theoretical analysis, *Journal of Political Economy*, Supplement, 70: 9 – 49.
5. Bulavsky, V.A., Isac, G., and Kalashnikov, V.V. (1998), Application of topological degree theory to complementarity problems. – In: *Multilevel Optimization: Algorithms and Applications*, A. Migdalas, P.M. Pardalos and P. Värbrand (Eds.), Dordrecht/Boston/London: Kluwer Academic Publishers. – P. 333 – 358.
6. Bulavsky, V.A., and Kalashnikov, V.V. (1994), One-parametric driving method to study equilibrium. *Economics and Mathematical Methods* (Ekonomika i Matematicheskie Metody, in Russian), 30: 129—138.
7. Bulavsky, V.A., and Kalashnikov, V.V. (1995), Equilibria in generalized Cournot and Stackelberg models. *Economics and Mathematical Methods* (Ekonomika i Matematicheskie Metody, in Russian), 35: 164--176.
8. Daly, V., and Ghatak, S. (1999), East-West European migration: Questions and some answers, paper presented at the Conference "*Migration and Mobility: The European Context*" at Kingston University, UK.
9. Figuières, C., Jean-Marie, A., Quérou, N., and Tidball, M. (2004), *Theory of Conjectural Variations.*New Jersey/London/Singapore/Shanghai/Hong Kong/Taipei/Bangalore: World Scientific.
10. Harris, J.R., and Todaro, M.P. (1970), Migration, unemployment and development: A two sector analysis, *American Economic Review* 60: 126 – 142.
11. Hatton, T.J. (1995), A model of UK emigration, 1870 – 1913, *Review of Economics and Statistics* 77: 407 – 415.
12. Hicks, J. (1932), *The Theory of Wages*, London: Macmillan.
13. Hugo, G.L. (1981), Village-community ties, village norms, and ethnic and social networks: A review of evidence from the third world. – In: De Long, G.D., and Gardner, R.W. (Eds.), *Migration Decision Making: Multidisciplinary Approaches to Micro-Level Studies in Developed and Developing Countries.* New York: Pergamon Press.
14. Isac, G., Bulavsky, V.A., and Kalashnikov, V.V. (2002), *Complementarity, Equilibrium, Efficiency and Economics*, Dordrecht/Boston/London: Kluwer Academic Publishers. – 468 pp.

15. Kalashnikov, V.V., and Kalashnikova, N.I. (2004), Demand and equilibrium in a network of oligopolistic markets. *Journal of Business and Economics Research*, 2: 51 – 64.
16. Kalashnikov, V.V., and Kalashnikova, N.I. (2006), Simulation of a conjectural variations equilibrium in a human migration model. *The International Journal of Simulation: Science, Systems & Technologies, The UK Simulation Society, Nottingham, UK* , 7, No. 9: 26 – 49.
17. Kinderlehrer, D., and Stampacchia, G. (1980), *An Introduction to Variational Inequalities and Their Applications.* New York: Academic Press.
18. Massey, D.S. (1990a), The social and economic origins of immigration, *Annals of the American Academy of Political and Social Sciences,* 510: 60 – 72.
19. Massey, D.S. (1990b), Social structure, household strategies, and the cumulative causation of migration, *Population Index,* 56: 1 – 26.
20. Massey, D.S., and Espana, F.G. (1987), The social process of international migration, *Science,* 237: 733 – 738.
21. Mincer, J. (1978), Family migration decisions, *Journal of Political economy*, 86: 749 – 773.
22. Nagurney, A. (1989), Migration equilibrium and variational inequalities, *Economics Letters,* 31: 109 – 112.
23. Nagurney, A. (1990), A network model of migration equilibrium with movement costs. *Mathematical and Computer Modelling,* 13: 79 – 88.
24. Nagurney, A., Jie Pan, and Lan Zhao (1992), Human migration network, *European Journal of Operational Research,* 59: 262 – 274.
25. Nagurney, A. (1999), *Network Economics: a Variational Inequality Approach* (revised 2nd edition), Dordrecht/Boston/London: Kluwer Academic Publishers.
26. Poirine, B. (1997), A theory of remittances as an implicit family loan arrangement, *World Economy,* 25: 589 – 611.
27. Ravenstein, E. (1889), The laws of migration, *Journal of the Statistical Society,* 52: 214 – 301.
28. Sandell, S.H. (1977), Women and the economics of family migration, *Review of Economics and Statistics,* 53: 157 – 163.
29. Schultz, T.W. (1961), Investment in human capital, *American Economic Review,* 51: 1 – 17.
30. Sjaastad, L.A. (1962), The costs and returns of human migration, *Journal of Political Economy,* 70: 80 – 93.
31. Smith, A. (1776), *Inquiry into the Nature and Causes of the Wealth of Nations*, London: Strahan/Cadell.
32. Stark, O. (1991), *The Migration of Labour.* Oxford: Basil Blackwell.
33. Stark, O., and Taylor, J.E. (1989), Relative deprivation and international migration, *Demography,* 26: 1 – 14.